TEMPERATURE DIFFERENCE ACROSS PHASE INTERFACE IN NUCLEATE BOILING

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Inzhenerno-Fizicheskii Zhurnal, Vol. 14, No. 3, pp. 430–436, 1968

UDC 536.423.1

An approximate method is presented for the determination of the temperature difference across the liquid-vapor phase interface in a vapor bubble, during boiling in a large volume.

Particular attention is being devoted in recent times to the problems of the mechanism of boiling, in connection with the development of new technology and in view of the fact that many aspects of this development have not yet been subjected to quantitative evaluation.

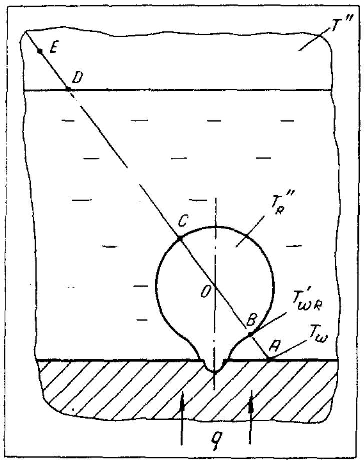

Fig. 1. Standard physical model.

Among the factors which have not been studied in great detail we find the magnitude of the discontinuity in the temperature ϑ at the boundary of separation between a superheated liquid and the vapor space of a bubble in boiling within a large volume. We will restrict the stated problem to an examination of the difference between the temperatures ϑ observed between the surface of the wall and the vapor space of the bubbles which have not yet separated from the heated surface.

Experimental data show that the vapor bubbles appear first at the microscopic roughness elements of the heating surface. The most probable sites for the formation of the vapor phase are the microscopic depressions [1].

The widely held opinion that microscopic protuberances served as such foci was not confirmed [2]. The liquid in the wall layer must be superheated relative to the saturation temperature for the vapor over the flat surface, i.e., $T_f > T''$.

It is our opinion that we can assume that the generation of vapor bubbles in an active hemispherical depression occurs in the following manner.

When a vapor bubble separates from the surface it intensifies the turbulization of the liquid within the depression and promotes the influx of colder liquid volumes from the main core. The liquid temperature within the depression drops to T_{0f}, which is the temperature at the beginning of the period referred to by Hsu [3] as the "waiting period" τ_0, which is the time needed to heat the liquid within the depression to T_f. When the heating is stopped, the heat flow is directed toward the center of the sphere. Because of the smallness of the volume of the system, the transfer of energy is accomplished by heat conduction which involves some inertia. The depression can be treated as a portion of a sphere with uniformly distributed heat sources and a sink of the total heat flow at the center of the sphere when the liquid is heated at an instant that is close to the steady-state value (i.e., the equilibrium value; this state is not attained, since the process proceeds without equilibrium within a fraction of a second). The calculations of [10] demonstrate that the temperature at the center may rise. Consequently, the particular shape of the depression (of the hemisphere), the nonsteadiness of the heating, and the presence of inertia set up conditions favorable to the concentration of molecules with elevated energy at the center of the depression, i.e., favorable to the elevation of temperature within this region. The pressure in the system is constant; there is therefore no further superheating of the liquid, and we have a new phase—a vapor phase. This phenomenon can be ex-

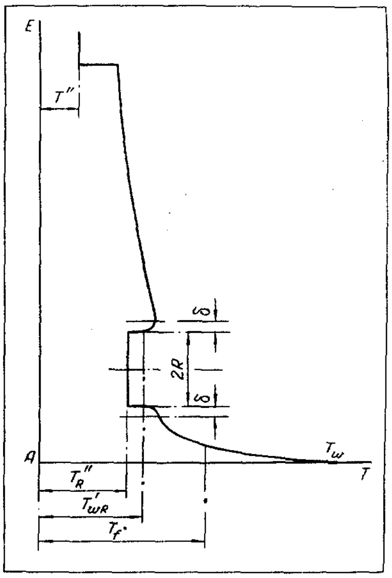

Fig. 2. Scheme of temperature field along beam AE; δ is the thickness of boundary thermal layer around gas bubble, according to [12].

plained in the following manner. Because of the above-noted factors, a nonuniform temperature field appears at the center of the sphere. This state of the system is unstable. The evening out of the temperature field occurs in the subsequent period (τ_{rel}). The minimum possible relaxation time can be defined as the time for the propagation of elastic waves within the liquid:

$$\tau_{\mathrm{rel}} = \frac{R_{\mathrm{cr}}}{w}. \tag{1}$$

For water $w \approx 1500$ m/sec, while for $p'' = 98.1$ bar, $R_{cr} = 0.12 \cdot 10^{-6}$ m.

Under these conditions $\tau_{rel} = 0.8 \cdot 10^{-10}$ sec.

According to [4], the thermodynamic properties of the substance may be affected by these fluctuations, whose period of existence is greater by an order than the quantity h/kT, i.e.,

$$\tau_{\mathrm{rel}} > \frac{h}{kT}. \tag{2}$$

For the conditions under consideration

$$\frac{h}{kT} = \frac{6.62 \cdot 10^{-34}}{1.38 \cdot 10^{-23} \cdot 582.5} = 0.82 \cdot 10^{-13}\mathrm{sec}\ .$$

Hence we can see that the time during which temperature fluctuations are present at the center of the sphere is sufficiently long to alter the thermodynamic properties of the substances within this region (i.e., to achieve conversion of liquid into vapor). It can be demonstrated that the above-described scheme for the appearance of vapor nuclei in real liquids proceeds in the presence of additional conditions: a solid particle must be present at the center of the sphere at the instant at which fluctuations in density appear (in industrial liquids, large amounts of such particles are present), or a gas bubble has to be present.

However, regardless of the conditions for the generation of the vapor, as demonstrated by experiment, in normal boiling we find the formation of vapor bubbles at the heating surface. Beginning our analysis from this confirmed fact, let us determine the magnitude of the temperature difference across the interface between the super-heated liquid and the vapor space of the bubble in the case of boiling within a large volume on a horizontal heating surface. The generated vapor bubble exhibits a finite radius R_{cr}. During the initial period of growth in τ_1, the bubble is close to spherical in shape (Fig. 1). Prior to separation from the surface, the bubble assumes a mushroom shape. It has been established in [5] that the rate of growth for the vapor bubble reaches a maximum in the initial period τ_1. For the boiling process at atmospheric pressure [6]

$$\tau_1 \approx 0.25\,\tau_{\mathrm{sep}}. \tag{3}$$

During this period the bubble grows from d_{cr} to d_1,

$$d_1 = 0.7 - 0.75\,d_{\mathrm{sep}}. \tag{3a}$$

Let us determine ϑ precisely during the period τ_1 in which the bubble is attached to the heating surface at one point.

The difference in ϑ of interest to us is a component part of the curve for the temperature field of the entire system. To derive a more general curve for the temperatures, the distribution of this quantity along the ray AE (according to Fig. 1) is established in Fig. 2, and we find that the aforementioned ray passes through the center of gravity for the vapor bubble without coinciding with the vertical.

We will assume the temperature T''_R within the vapor bubble to be approximately identical throughout the entire volume; T'_{wR} denotes the temperature of the liquid surrounding the bubbles. The pressure p''_R of the saturated vapor in the bubble is uniquely associated with the temperature T''_R of the saturated vapor. A necessary condition for the generation and growth of the vapor bubble is the superheating of the liquid relative to the temperature T' of the saturated vapor above the flat surface [7] to a temperature $T_f = T'_{wR}$.

We know that the equilibrium coexistence of the vapor and liquid phases is movable, i.e., the number of molecules leaving the liquid is equal to the number of molecules absorbed by the liquid surface. The process of vapor production in boiling is without equilibrium. The apparent generation of vapor is caused in this case by the fact that the number of molecules remaining within the vapor space is larger than the number of molecules absorbed by the liquid. According to kinetic theory, a temperature jump occurs at the boundary of separation between the phases in this case (Fig. 2). The magnitude of this jump can be determined by means of the condensation coefficient. The latter serves to evaluate the number of molecules captured by the liquid. It can range in value from zero to unity.

We will treat the vapor as an ideal gas, and we will assume the processes of molecule release from the liquid and of molecule absorption to be independent of each other. On the basis of concepts from kinetic theory [8], for vapor condensation we then have

$$j = \frac{2k}{2-k}\left(\frac{p''}{\sqrt{2\pi R''T''}} - \frac{p''_w}{\sqrt{2\pi R''T_w}}\right). \tag{4}$$

We will assume in approximate terms that (4) is also valid for evaporation from a curvilinear surface. The process of vapor formation in boiling is always accompanied by the transition of some of the molecules to the liquid.

According to [9], the intensity of vapor formation can therefore be calculated from the condensation formula

$$\frac{2k}{2-k}\left(\frac{p''_R}{\sqrt{2\pi R''T''_R}} - \frac{p''_{wR}}{\sqrt{2\pi R''T'_{wR}}}\right) = j. \tag{5}$$

Neglecting the work of the forces of surface tension and adhesion, we can assume that all of the heat transmitted by the heating surface represents the heat of phase conversion

$$q = rj,$$

whence

$$j = \frac{q}{r}. \tag{6}$$

We can express the density of the heat flow in terms of the rate of growth for the vapor bubble [5]:

$$q = r\rho'' \frac{dR}{d\tau}. \tag{7}$$

On the basis of (6) and (7), for the intensity of vapor formation we can write

$$j = \rho'' \frac{dR}{d\tau}. \tag{8}$$

Let us substitute this expression into (5):

$$\frac{2k}{2-k}\left(\frac{p''_R}{\sqrt{2\pi R'' T''_R}} - \frac{p''_{wR}}{\sqrt{2\pi R'' T'_{wR}}}\right) = \rho'' \frac{dR}{d\tau}. \tag{9}$$

The unknown in Eq. (9) is the magnitude of the vapor temperature T''_R within the bubble. The pressure p''_R of the saturated vapor within the bubble is uniquely associated with the temperature T''_R and can be determined from tables listing values for the saturated vapor.

Having specified the value of the temperature $T_f = T'_{wR}$ for the superheating of the liquid, and having used the tables to determine the saturation pressure corresponding to T'_{wR}, we can calculate the vapor temperature from (9) in a bubble of radius R. However, since Eq. (9) also includes the pressure p''_R of the saturated vapor, which is associated with the unknown T''_R, the calculation of the latter is accomplished by the method of successive approximations, i. e., initially we assume a tentative magnitude for the vapor temperature T''_R within the bubble, then we take the saturation pressure p''_R corresponding to T''_R from the tables, and finally, the sought temperature T''_R is calculated from Eq. (9). The calculation is concluded as soon as the initially adopted temperature value coincides with the found value. The value of T''_R is determined most easily for the initial period τ_1 of vapor-bubble growth. The time-variation of the bubble diameter in the initial period follows a law that is close to the linear [5, 6]. It may therefore be assumed in approximate terms that for the period τ_1 the constancy of the first derivative is characteristic:

$$\frac{dR}{d\tau} = \text{const.}$$

According to [10], the time for the bubble growth is

$$\tau_{\text{sep}} = \frac{r\rho''}{8\lambda\Delta T_{\text{super}}} d_{\text{sep}} \left[1 - \left(\frac{d_{\text{cr}}}{d_{\text{sep}}}\right)^2\right]. \tag{10}$$

The quantity d_{sep} is determined from a familiar formula; if we know τ_{sep}, with (3) and (3a), we can find the values of τ_1 and R_1. Consequently, the rate of bubble growth in the first period is defined as

$$\frac{dR}{d\tau} = \frac{R_1}{\tau_1}. \tag{11}$$

The quantity ΔT_{super} in formula (10) is understood to refer to the ratio between the superheating of the liquid surrounding the vapor bubble and the temperature T" of the saturated vapor above the flat surface at a specified pressure in the vessel (we neglect the hydrostatic pressure), i. e., $\Delta T_{\text{super}} = T_f - T''$ or, what amounts to the same,

$$\Delta T_{\text{super}} = T'_{wR} - T''. \tag{12}$$

For the initial period τ_1 Eq. (9) thus enables us, in approximate terms, to calculate the magnitude of the temperature jump

$$\vartheta = T'_{wR} - T''_R, \tag{13}$$

which occurs at the liquid-vapor interface in the bubble in the thin wall layer at the heating surface. The solution of Eq. (9), with the aid of seven-place logarithmic tables prepared by the author for water, demonstrate that the quantity ϑ at the interface is small for the above-described conditions. Thus, for $p'' = 0.981 \cdot 10^5$ N/m² and $\Delta T_{\text{super}} = 5°$ K, ϑ assumes a value equal to 0.015° K, while for $p'' = 98.1 \cdot 10^5$ N/m² and $\Delta T_{\text{super}} = 25°$ K, $\vartheta = 0.01°$ K.

For the second case, here we assume in approximate terms that (3) is valid. To determine the condensation coefficient we turned to reference [11].

In the first example k = 0.54 when $T''_R = 378.135°$ K. In the second example k = 0.712 when $T''_R = 607.67°$ K.

The temperature of the vapor within the bubbles cannot be measured experimentally at the present time. However, there exists an indirect method of determining the accuracy of the results obtained. When the vapor bubble is floating, it increases in volume, absorbing heat from the superheated liquid. Relative to the temperature of the saturated vapor over the flat surface, the superheating of the liquid is determined from the familiar formula

$$\Delta T_{\text{super}} = T'_{wR} - T'' = \frac{2\sigma T''}{r\rho'' R}. \tag{14}$$

With an increase in the radius, the medium need not be as intensively superheated for vapor bubbles to exist within it. A vapor bubble leaving the liquid expands in the limit to $R = \infty$, so that $\Delta T_{\text{super}} \to 0$, and for the limit case the liquid temperature becomes equal to the vapor temperature T". Consequently, the absolute equilibrium magnitude of the vapor temperature within the bubble in the upper layers of the liquid is smaller than in the lower layers. Simplifying the actual processes somewhat, we see that when the bubble begins to float (thus increasing in volume) a vapor mixture begins to form within the bubble.

The components include liquid vapors of various temperatures. If we know the volume of the vapor within the bubble at the instant of its separation from the heating surface and on leaving the surface of the liquid, we can calculate the temperature of the mixture. We can use Eq. (9) to determine the vapor temperature within a bubble whose diameter has reached the size required for separation. For this we have to use the average integral value of T'_{wR} of the surrounding liquid. In first approximation, assuming the temperature of the last vapor portions formed prior to departing the liquid surface to be equal to T", we can tentatively derive the average actual vapor temperature which should be found in a vapor bubble of radius R near the evaporation surface (in a liquid). According to our calculations, for the conditions of the Yakob experiment [13], this temperature is equal to 373.75°K (100.6° C). Consequently, the surrounding liquid should exhibit the temperature

$$T'_{wR} = T''_R + \vartheta.$$

In the case of water, for $p'' = 0.981$ bar, $\vartheta = 0.015$°K. Extending the data from the solution of Eq. (9) to the upper layers of the liquid, we find that

$$T_f = T'_{wR} = 373.75 + 0.015 = 373.765^\circ K.$$

From the data of [13], in the upper layers of the liquid we have $T_f = 373.55$° K (100.4° C). For approximate calculations, the derived results may be regarded as sufficiently accurate. Generalizing the results of this work, we should point out that the "apparent" magnitude of the temperature difference for the liquid-vapor system is of a different order of magnitude. In the wall layer at the heating surface it is estimated at 0.01–0.015° K for water. In the upper layers of the liquid we have $\vartheta \approx 0.4$–0.6° K (for a height of 0.06 m).

NOTATION

T_f is the instantaneous temperature of the boiling fluid; τ_{rel} is the relaxation time; R_{cr} and R are the critical and instantaneous radii of the vapor bubble; h is the Planck constant; k is the Boltzmann constant and condensation coefficient; j is the intensity of "apparent" condensation and intensity of "apparent" vapor generation, kg/m^2 · sec; τ_1 and τ_{sep} are the initial period of vapor-bubble growth and the period of its growth up to d_{sep}; R" is the gas constant for the vapor; w is the velocity of elastic-wave propagation; p''_W is the pressure of the saturated vapor at the surface temperature (of the condensate film); R_1 is the radius of the vapor bubble at the end of the initial period τ_1; p''_{WR} is the pressure of the saturated vapor at a liquid temperature T'_{WR} on surface of the vapor bubble with radius R; p''_R is the pressure of the saturated vapor in the vapor bubble with radius R at vapor temperature T''_R; T'_{WR} is the wall temperature of the vapor bubble with radius R (liquid temperature on bubble surface); T''_R is the mean temperature of the saturated vapor in the bubble; q is the density of heat flux; r is the heat of vapor generation; d_{cr}, d_1, and d_{sep} are the vapor bubble diameters: minimum—corresponding to initial period separation; ρ'' is the density of the saturated vapor; λ is the thermal conductivity of the liquid.

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9 March 1967

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